CIRCLE

EXERCISE – I HINTS & SOLUTIONS

Sol.1 Intersection of given lines is centre

$2x - 3y - 5 = 0$

$3x - 4y - 7 = 0$

$$\frac{x}{21-20} = \frac{y}{-15+14} = \frac{1}{-8+9}$$

$\Rightarrow\ x = 1,\ y = -1$

$(1, -1),\ \pi r^2 = 154 \Rightarrow r^2 = \dfrac{154}{22} \times 7$

$\Rightarrow\ r = 7$

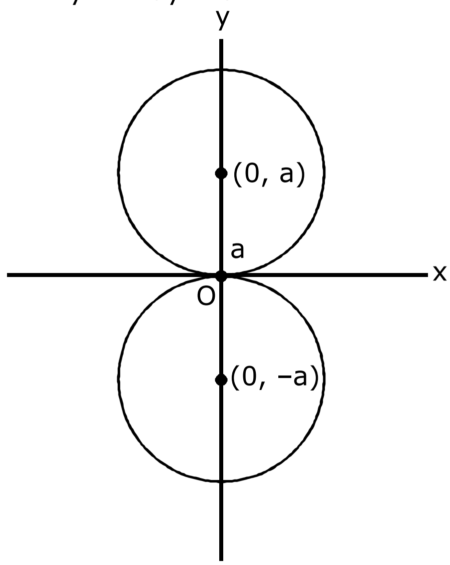
$g = -1,\ f = 1,\ c = g^2 + f^2 - r^2$

$= 1 + 1 - 49 = -47$

$x^2 + y^2 - 2x + 2y - 47 = 0$

Sol.2 $x^2 + (y \pm a)^2 = a^2$

$x^2 + y^2 \pm 2ay = 0$



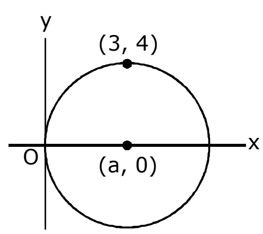
Sol.3 Let centre (a, 0), radius = a

$(a - 3)^2 + 4^2 = a^2$

$-6a + 9 + 16 = 0$

$6a = 25 \Rightarrow\ a = \dfrac{25}{6}$

$g = -\dfrac{25}{6}$, f=0, c=0



$x^2 + y^2 - \dfrac{25}{3}x = 0$

Aliter :

c = 0, f = 0 Let circle

$x^2 + y^2 + 2gx = 0$ passes (3, 4)

$9 + 16 + 6g = 0$

$g = \dfrac{-25}{3} \Rightarrow 3(x^2 + y^2) - 25x = 0$

Sol.4 Centre (2, –1), radius $= \sqrt{(3-2)^2 + (6+1)^2}$

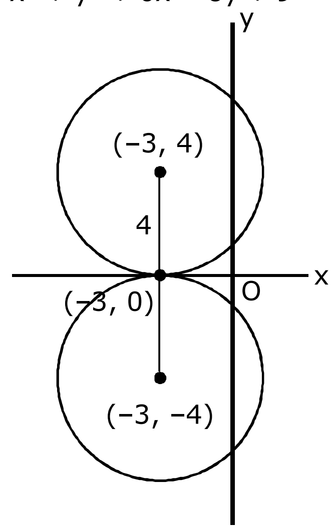
$= \sqrt{1+49} = \sqrt{50}$

$(x - 2)^2 + (y + 1)^2 = 50$

$x^2 + y^2 - 4x + 2y - 45 = 0$

Sol.5 $(x + 3)^2 + (y \pm 4)^2 = 16$

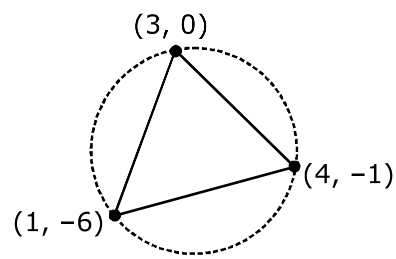
$x^2 + y^2 + 6x \pm 8y + 9 = 0$



Sol.6 Let the centre (a, b)

$(a - 3)^2 + (b)^2 = (a - 1)^2 + (b + 6)^2$

$= (a - 4)^2 + (b + 1)$



(i) & (ii)

$-6a + 9 = -2a + 1 + 12b + 36$

$\Rightarrow 4a + 12b + 28 = 0 \Rightarrow a + 3b + 7 = 0$

(i) & (iii)

$-6a + 9 = -8a + 16 + 2b + 1$

$\Rightarrow 2a - 2b = 8 \qquad \Rightarrow a - b = 4$

$a = \dfrac{5}{4},\ b = -\dfrac{11}{4} \quad r = \sqrt{\dfrac{49}{16} + \dfrac{121}{16}} = \dfrac{\sqrt{170}}{4}$

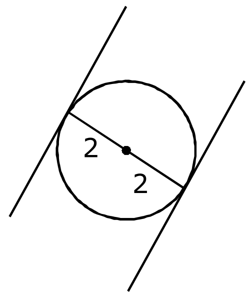
$g = -\dfrac{5}{4},\ f = \dfrac{11}{4},\ c = \dfrac{25}{16} + \dfrac{121}{16} - \dfrac{170}{16}$

$= \dfrac{-24}{16} = \dfrac{-3}{2}$

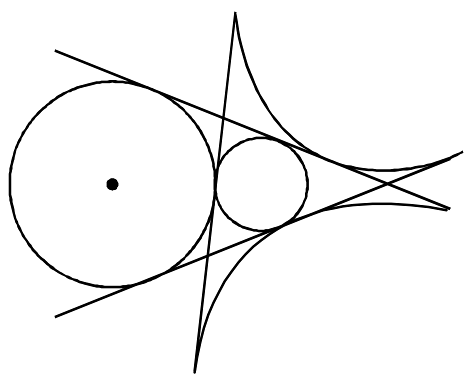
$x^2 + y^2 - 2 \cdot \frac{5}{4}x + 2 \cdot \frac{11}{4}y - \frac{3}{2} = 0$

$2x^2 + 2y^2 - 5x + 11y - 3 = 0$

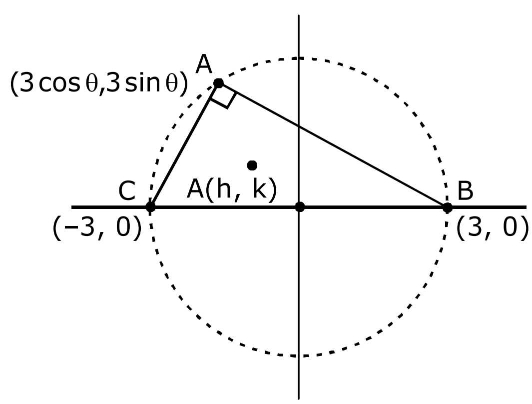
Sol.7 $4 = \left|\frac{c_1 - c_2}{\sqrt{1+3}}\right| \Rightarrow |c_1 - c_2| = 8$



Sol.8 Four circles
{one incircle & three excircles}



Sol.9 Circle is
$x^2 + y^2 = 9$
$\therefore$ co-ordinate of point
A ($3\cos\theta$, $3\sin\theta$)

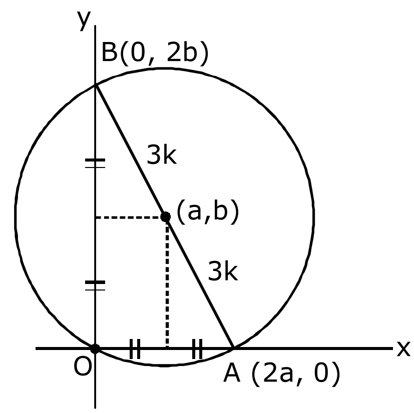


centroid of ΔABC is P(h, k) whose coordinate is

$$\left(\frac{3 + 3\cos\theta - 3}{3}, \frac{0 + 0 + 3\sin\theta}{3}\right) \equiv (\cos\theta, \sin\theta)$$

$h = \cos\theta$, $k = \sin\theta$
$h^2 + k^2 = 1 \Rightarrow x^2 + y^2 = 1$

Sol.10 Let centre (a, b)
$AB^2 = (6k)^2 = (2a)^2 + (-2b)^2$
$\Rightarrow a^2 + b^2 = 9k^2$



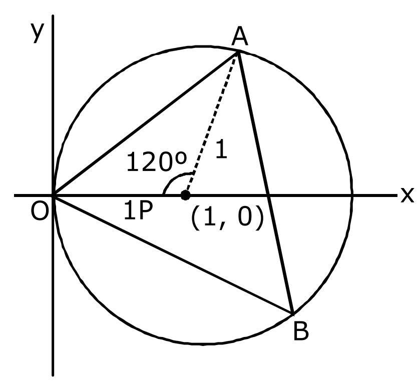
Let centroid of ΔOAB is (x_1, y_1)

$x_1 = \frac{2a}{3}$, $y_1 = \frac{2b}{3} \Rightarrow a = \frac{3}{2}x_1$, $b = \frac{3}{2}y_1$

$\Rightarrow \left(\frac{3x_1}{2}\right)^2 + \left(\frac{3y_1}{2}\right)^2 = 9k^2$

$\Rightarrow x_1^2 + y_1^2 = (2k)^2 \Rightarrow x^2 + y^2 = (2k)^2$

Sol.11 $x^2 + y^2 - 2x = 0$
$(x - 1)^2 + y^2 = 1$
area $\Delta OAB = 3$ or $\Delta(OAP)$



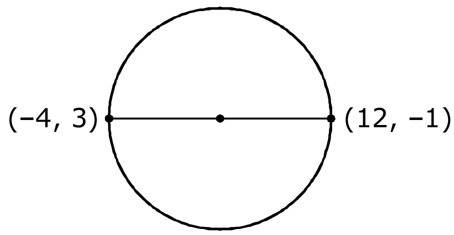
$= 3 \times \frac{1}{2} 1.1 \sin 120º$

$= \frac{3}{2}\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$ sq. units

Sol.12 $(x + 4)(x - 12) + (y - 3)(y + 1) = 0$

$x^2 + y^2 - 8x - 2y - 51 = 0$

$f = (-1), c = -51$

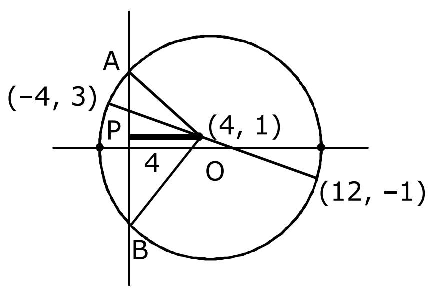


y intercept $= 2\sqrt{f^2 - c} = 2\sqrt{1+51}$

$= 2\sqrt{52} = 4\sqrt{13}$

Aliter

centre (4, 1), radius = $\sqrt{68}$



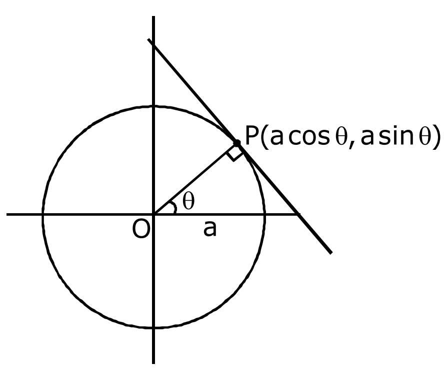
AP = $\sqrt{68-16} = \sqrt{52}$

AB = 2(AP) = $2\sqrt{52} = 4\sqrt{13}$

Sol.13 $x^2 + y^2 = a^2$

$m_N = \tan\theta$

$m_T = -\frac{1}{m_N} = \frac{1}{\tan\theta} = -\cot\theta$

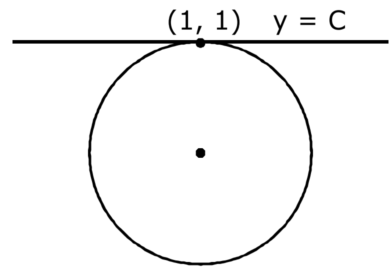


Sol.14 $\ell x + my + n = 0, x^2 + y^2 = r^2$

$$r = \left|\frac{n}{\sqrt{\ell^2 + m^2}}\right| \Rightarrow r^2 (l^2 + m^2) = n^2$$

Sol.15 $x^2 + y^2 - 2x + 2y - 2 = 0$

Tangent at (1, 1)



$x + y - (x + 1) + (y + 1) - 2 = 0$

$y - 1 + y + 1 - 2 = 0$

$2y - 2 = 0$

$y = 1 \Rightarrow c = 1$

Sol.16 Tangent at (x_1, y_1) is

$xx_1 + yy_1 = 25$

$3x + 4y = 25 \Rightarrow x_1 = 3, y_1 = 4$ (3, 4)

Sol.17 Let tangent from (0, 1) on $x^2 + y^2 - 2x + 4y = 0$

$y - 1 = mx$ $\quad$ C(1, –2), r = $\sqrt{5}$

$\Rightarrow mx - y + 1 = 0$

$$r = \sqrt{5} = \frac{|m+2+1|}{\sqrt{m^2+1}} \Rightarrow 5(m^2+1) = (m+3)^2$$

$\Rightarrow 4m^2 - 6m - 4 = 0 \Rightarrow 2m^2 - 3m - 2 = 0$

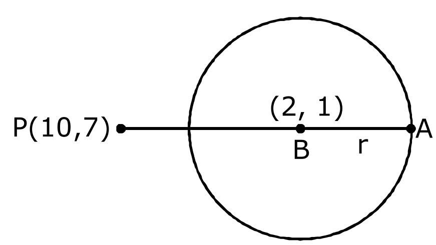
$\Rightarrow (m - 2)(2m + 1) = 0 \Rightarrow m = 2, -\frac{1}{2},$

Tangents are $2x - y + 1 = 0$

$x + 2y - 2 = 0$

Sol.18 $x^2 + y^2 - 4x - 2y - 20 = 0$, P (10, 7)

$S_1 = 100 + 49 - 40 - 14 - 20 > 0$



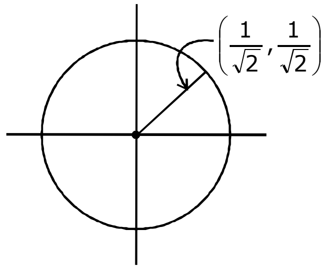
P lies outside

O (2, 1), r = $\sqrt{4+1+20} \Rightarrow r = 5$

greatest distance = PA = PO + OA

$= \sqrt{8^2 + 6^2} + 5 = 10 + 5 = 15$

Sol.19 Normal is diameter passing through centre (0, 0)

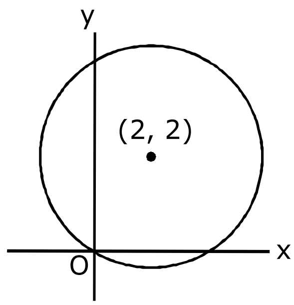


$$\& \ m = \frac{\frac{1}{\sqrt{2}} - 0}{\frac{1}{\sqrt{2}} - 0} = 1$$

$y = x \Rightarrow \ x - y = 0$

Sol.20 $x^2 + y^2 - 4x - 4y = 0$

$C(2, 2),\ r = \sqrt{4+4-0} = 2\sqrt{2}$



Parametric Coordinate

$(2 + 2\sqrt{2}\cos\theta,\ 2 + 2\sqrt{2}\sin\theta)$

Sol.21 $2(x^2 + y^2) - 7x + 9y - 11 = 0$, P (2, 3)

Point lie outside

$\therefore\ x^2 + y^2 - \frac{7}{2}x + \frac{9}{2}y - \frac{11}{2} = 0$

Length of tangent

$T_1 = \sqrt{s_1} = \sqrt{4 + 9 - 7 + \frac{27}{2} - \frac{11}{2}}$

$= \sqrt{6+8} = \sqrt{14}$

Sol.22 Pair of tangents from (0, 0) on

$x^2 + y^2 + 20(x + y) + 20 = 0$

$T^2 = SS_1$

$(0 + 20(x + y) + 20)^2$

$= (x^2 + y^2 + 20x + 20y + 20)(20)$

$(x + y)^2 + 400(x + y) + 400$

$= 20(x^2 + y^2) + 400(x + y) + 400$

$5(x + y)^2 = x^2 + y^2$

$4x^2 + 4y^2 + 10xy = 0$

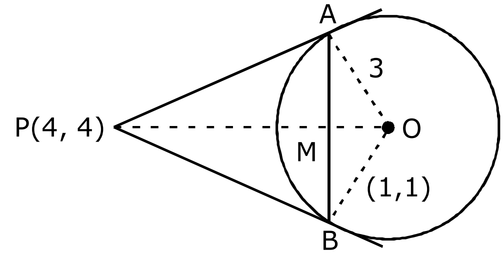
$2x^2 + 5xy + 2y^2 = 0$

M-II C.O.C from (0, 0) 8 honoziniation to circle and get pair to tangents.

Sol.23 $x^2 + y^2 - 2x - 2y - 7 = 0$

$0(1, 1),\ r = \sqrt{1+1+7} = 3$

Equation of AB



$4x + 4y - (x + 4) - (y + 4) - 7 = 0$

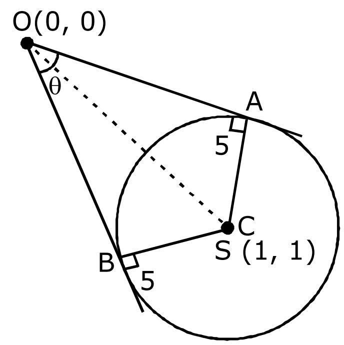
$3x + 3y = 15 \Rightarrow\ x + y = 5$

$OM = \frac{|1+1-5|}{\sqrt{1^2+1^2}} = \frac{3}{\sqrt{2}}$

$AM = \sqrt{3^2 - \frac{3^2}{2}} = \frac{3}{\sqrt{2}} \Rightarrow AB = 2.\frac{3}{\sqrt{2}} = 3\sqrt{2}$

Sol.24 $(x - 7)^2 + (y + 1)^2 = 25$ tangents (0, 0)

$C(7, 1),\ r = 5$



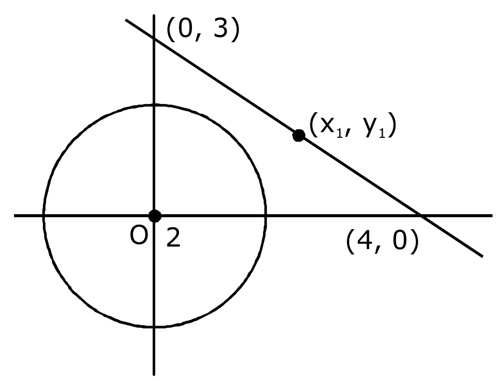
$OA = T_1 = \sqrt{S_1} = \sqrt{50-25} = 5$

$OA = AC = 5$

$\angle AOC = \frac{\pi}{4}$ $\qquad \angle AOB = \theta = \frac{\pi}{2}$

Sol.25 equation of pair of tangents and find angle betwen time.

$x^2 + y^2 = 4$ & line $3x + 4y = 12$



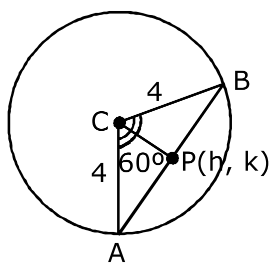
Let P (x_1, y_1) oin given line & C.O.C of P.
$xx_1 + yy_1 = 4$ ....(i) P satisfy given line
$3x_1 + 4y_1 = 12$ ....(ii)
3(i) – (ii)

$$\Rightarrow \quad 3xx_1 + 3yy_1 = 12$$
$$3x_1 \pm 4y_1 = 12$$
$$- \quad - \quad -$$
$$3x_1(x) + y_1(3y - 4) = 0$$
$$(x - 1) + \lambda(3y - 4) = 0$$
$$\Rightarrow \quad L_1 + \lambda L_2 = 0$$

Find point $x = 1$ & $y = \frac{4}{3} \Rightarrow \left(1, \frac{4}{3}\right)$

Sol.26 Let mid point of cord P(h, k)
$x^2 + y^2 - 2x - 4y - 11 = 0$
C(1, 2), r = 4

$CP = 4\cos 30^\circ = 4\,\frac{\sqrt{3}}{2} = 2\sqrt{3}$



We know that locus is circle
whose radius is CP & centre (1, 2)

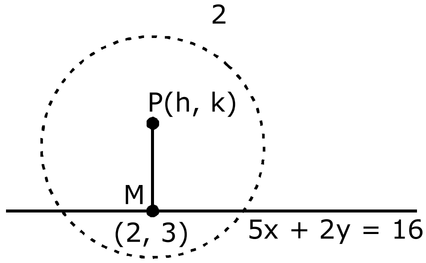
$(x - 1)^2 + (y - 2)^2 = (2\sqrt{3})^2$

$\Rightarrow x^2 + y^2 - 2x - 4y - 7 = 0$

M-II equation of chord $T = S_1$ have a distance from centre is $2\sqrt{3}$ and get the locus.

Sol.27 Let the centre P(h, k)

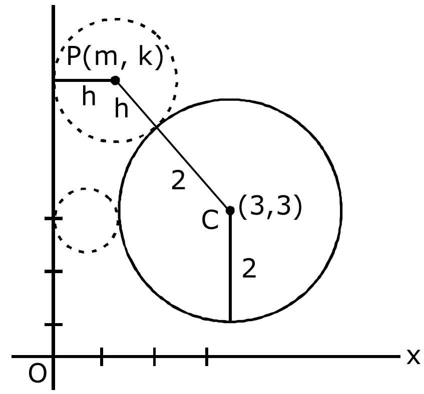
$$m_{PH} = \frac{-1}{m_2} = \frac{-1}{-\frac{5}{2}} = \frac{2}{5}$$



$$\frac{k-3}{h-2} = \frac{2}{5}$$

$2h - 5k + 11 = 0$
$2x - 5y + 11 = 0 \quad \rightarrow$ Line PM.

Sol.28 $x^2 + y^2 - 6x - 6y + 14 = 0$
centre (3, 3), radius = 2

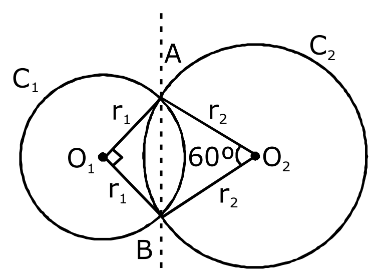


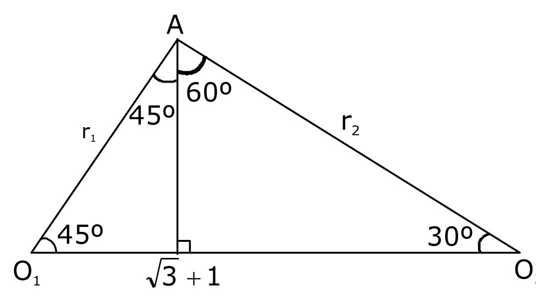
$\Rightarrow$ radius is h ($\because$ touches y-axis)
PC = h + 2

$\sqrt{(h-3)^2 + (k-3)^2} = (h + 2)$

$\Rightarrow h^2 + k^2 - 6h + 18 = 4 + 4h + h^2$
$\Rightarrow k^2 - 10h - 6k + 14 = 0$
$\Rightarrow y^2 - 10x - 6y + 14 = 0$

Sol.29 $O_1O_2 = \sqrt{3} + 1$
Sine rule in AO_1O_2





$$\frac{\sqrt{3}+1}{\sin 105^\circ} = \frac{r_1}{\sin 30^\circ} = \frac{r_2}{\sin 45^\circ}$$

$$r_1 = \frac{\sqrt{3}+1}{\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)} \times \frac{1}{2} = \sqrt{2}$$

$r_2 = 2$

Sol.30 Let the centre of cirlce (h_1k)

radius $r = \dfrac{\ell h + mk + n}{\sqrt{\ell^2 + m^2}}$

Equation $(x-h)^2+(y-k)^2 = \dfrac{(\ell h + mk + n)^2}{\ell^2 + m^2}$

& $x^2 + y^2 = 9$ cut orthogonally

$2g_1g_2 + 2f_1f_2 = c_1 + c_2 \qquad f_2 = g_2 = 0$

$\therefore\ C_1 + C_2$

$h^2 + k^2 - \dfrac{(\ell h + mk + n)^2}{\ell^2 + m^2} + (-9) = 0$

$\Rightarrow (x^2 + y^2 - 9) . (\ell^2 + m^2)$
$\qquad - (\ell x + my + n)^2 = 0$

Sol.31 $y^2 - 2y + 2xy = 0$ represent normals.

$\{(y (y - 2) - 2x (y - 2) = 0)$
$(y - 2) (y - 2x) = 0\}$

Intersection point is centre

$y = 2$ & $y = 2x \Rightarrow x = 1 , y = 2$

centre (1, 2), passing thorugh (2, 1)

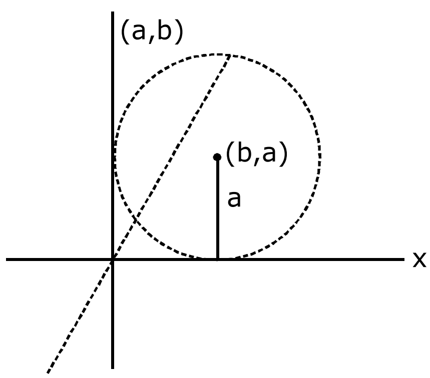
$r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

$(x - 1)^2 + (y - 2)^2 = 2$

$x^2 + y^2 - 2x - 4y + 3 = 0$

Sol.32 Reflection of (a, b) in $y - x = 0$ is (b, a)

centre (b, a) touching x-axis.



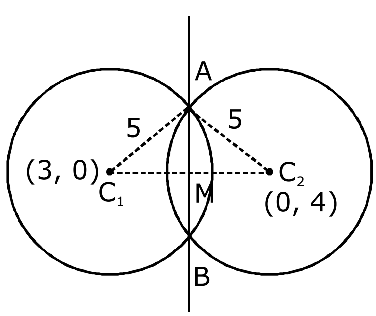
$r = Q$

$(x - b)^2 + (y - a)^2 = a^2$

$x^2 + y^2 - 2bx - 2ay + b^2 = 0$

Sol.33 Common chord $r_1 = 5 = r_2$

$-6x + 8y - 7 = 0 \quad \Rightarrow \quad 6x - 8y + 7 = 0$



$C_1M = \left|\dfrac{18 - 0 + 7}{\sqrt{6^2 + 8^2}}\right| = \dfrac{25}{10} = \dfrac{5}{2}$

$AM = \sqrt{25 - \dfrac{25}{4}} = \sqrt{\dfrac{75}{4}} = \dfrac{5}{2}\sqrt{3}$

$AB = 2AM = 5\sqrt{3}$

Aliter :

$r_1 = r_2 = 5$

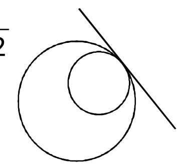
$AC_1 = AC_2 = C_1C_2 = 5$

$\Rightarrow \Delta AC_1C_2$ equilateral

$AM = 5\sin 60^\circ = \dfrac{5\sqrt{3}}{2} \Rightarrow AB = 5\sqrt{3}$

Sol.34 $S_1 \Rightarrow C_1(1, 0),\ r_1 = \sqrt{2}$

$S_2 \Rightarrow C_2\ (0, 1),\ r_2 = 2\sqrt{2}$



$C_1C_2 = \sqrt{1^2 + 1^2} = \sqrt{2}$

$C_1C_2 = |\, r_2 - r_1 \,|$

$\sqrt{2} = \sqrt{2}$

Internally touch $\therefore$ common tangent is one.

Sol.35 $S_1 - S_3 = 0 \Rightarrow 16y + 120 = 0$

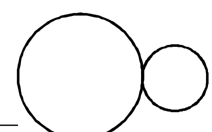
$\Rightarrow y = \dfrac{-120}{16} \Rightarrow \quad y = -\dfrac{15}{2} \quad \Rightarrow x = 8$

Intersection point of radical axis is

$\left(8, \dfrac{-15}{2}\right)$

Sol.36 $x^2 + y^2 = 9$

$\Rightarrow C_1(0, 0),\ r_1 = 3$

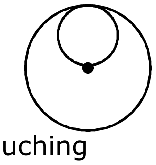


$x^2 + y^2 + 6y + c=0$

$C_2\ (0, -3),\ r_2= \sqrt{9 - c}$

If circle are externally touching

$c_1c_2 = r_1+r_2$



$B = 3 + \sqrt{9 - c}$

$\Rightarrow c = 9$

If cirlce are internally touching

$C_1C_2 = |r_1 - r_2|$

$3 = + 3 - \sqrt{9 - c}$ or $\quad 3 = - 3 + \sqrt{9 - c}$

$\Rightarrow c = 9 \qquad\qquad \Rightarrow 6 = \sqrt{9 - c}$

$\qquad\qquad\qquad \Rightarrow \quad c = - 27$

$c = 9, - 27$

Aliter :

Common tangent of S_1 & S_2

$6y + c + 9 = 0$

$$3 = \left|\frac{c+9}{\sqrt{6^2}}\right| \Rightarrow 18 = |c + 9|$$

$\Rightarrow c = 9, -27$

Sol.37 $x^2 + y^2 + 2g_1x + 2f_1y = 0$

$\Rightarrow C_1(-g_1, -f_1),\ r_1 = \sqrt{g_1^2 + f_1^2}$

$x^2 + y^2 + 2g_2x + 2f_2y = 0$

$\Rightarrow C_2(-g_2, -f_2),\ r_2 = \sqrt{g_2^2 + f_2^2}$

If externally touches $C_1C_2 = r_1 + r_2 = |r_1+r_2|$

If internally touches $C_1C_2 = |r_1-r_2|$

both $\Rightarrow C_1C_2 = |r_1 \pm r_2|$

$$\sqrt{(g_1-g_2)^2 + (f_1-f_2)^2} = \left|\sqrt{g_1^2+f_1^2} \pm \sqrt{g_2^2+f_2^2}\right|$$

$\Rightarrow (g_1 - g_2)^2 + (f_1 - f_2)^2 = (g_1^2 + f_1^2)$

$+ (g_2^2 + f_2^2) \pm 2\sqrt{g_1^2+f_1^2}\sqrt{g_2^2+f_2^2}$

$\Rightarrow g_1^2g_2^2 + f_1^2f_2^2 + 2g_1g_2f_1f_2 = g_1^2g_2^2 + f_1^2f_2^2 + g_1^2f_2^2 + g_2^2f_1^2$

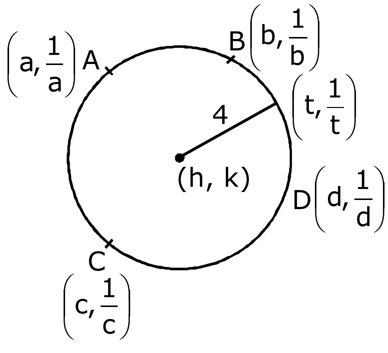
$\Rightarrow (g_1f_2)^2 + (g_2f_1)^2 - 2g_1g_2f_1f_2 = 0$

$\Rightarrow (g_1f_2 - g_2f_1)^2 = 0 \Rightarrow g_1f_2 = g_2f_1$

$$\Rightarrow \frac{f_1}{g_1} = \frac{f_2}{g_2}$$

Sol.38 Let the centre (h, k)

$$(h - t)^2 + \left(k - \frac{1}{t}\right)^2 = 16$$



$$\Rightarrow h^2 + k^2 - 2ht - 2\frac{k}{t} + t^2 + \frac{1}{t^2} = 16$$

$\Rightarrow t^4 - 2ht^3 + t^2(h^2 + k^2 - 16) - 2kt + 1 = 0$

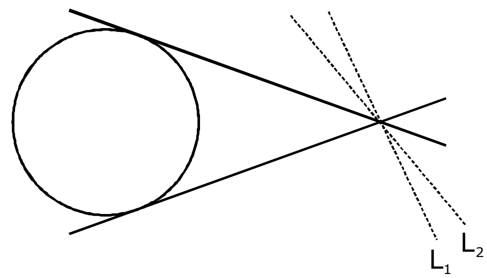
roots of this equation a, b, c, d

a.b.c.d = +1

Sol.39 Line poassing thorough the intesection points of L_1 & L_2 is tangent of circle

$(2x - 3y + 1) + \lambda(3x - 2y - 1) = 0$

$(2 + 3\lambda)x - y(3 + 2\lambda) + (1 - \lambda) = 0$ is tangent of given circle



centre (–1, 2), $r = \sqrt{1 + 2^2 - 0} = \sqrt{5}$

$$\sqrt{5} = \left|\frac{-(2+3\lambda) - 2(3+2\lambda) + (1-\lambda)}{\sqrt{(2+3\lambda)^2 + (3+2\lambda)^2}}\right|$$

$$= \frac{|-8\lambda - 7|}{\sqrt{(2+3\lambda)^2 + (3+2\lambda)^2}}$$

$\Rightarrow 5[(2 + 3\lambda)^2 + (3 + 2\lambda)^2] = (8\lambda + 7)^2$

$\Rightarrow 65\lambda^2 + 120\lambda + 65 = 64\lambda^2 + 112\lambda + 49$

$\Rightarrow \lambda^2 + 8\lambda + 15 = 0 \Rightarrow (\lambda + 4)^2 = 0$

$\Rightarrow \lambda = -4 \Rightarrow$ tangent $-10x + 5y + 5 = 0$

$\Rightarrow 2x - y - 1 = 0$

Aliter :

Point of intersection is (1, 1)

$2x - 3y + 1 = 0$

$3x - 2y - 1 = 0$

(1, 1) lies on circle

$\therefore$ tangent of circle is

$x \cdot 1 + y \cdot 1 + (x + 1) - 2y(y + 1) = 0$

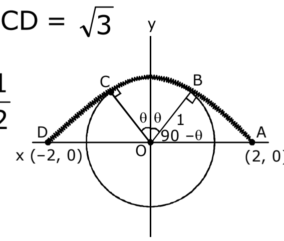
$2x - y - 1 = 0$

Sol.40 r = 1

$AB = \sqrt{2^2 - 1} = CD = \sqrt{3}$

$$\cos(90 - \theta) = \frac{1}{2}$$

$$\sin\theta = \frac{1}{2}$$

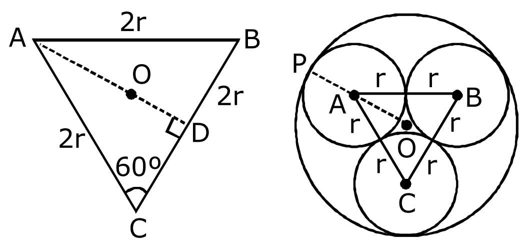


$\theta = \frac{\pi}{6} \Rightarrow 2\theta = \frac{\pi}{3}$

arc BC = $\ell(\widehat{BC}) = \frac{2\pi.1}{6} = \frac{\pi}{3}$

Shortest path is = $2\sqrt{3} + \frac{\pi}{3}$

Sol.41 AD = 2r sin 60º = $2r\frac{\sqrt{3}}{2} = \sqrt{3}\,r$



AO = $\sqrt{3}r \times \frac{2}{3} = \frac{2r}{\sqrt{3}}$

OP = OA + AP

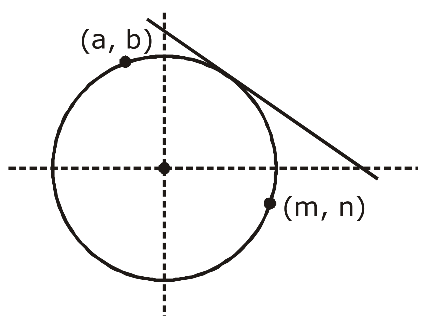
$= \frac{2r}{\sqrt{3}} + r = \frac{(2+\sqrt{3})r}{\sqrt{3}}$

Sol.42 A

Given $a^2 + b^2 = 1$, $m^2 + n^2 = 1$

i.e. points (a, b) & (m, n) on the circle

$x^2 + y^2 = 1$ tangent at (a, b)



ax + by – 1 = 0 point (0, 0) & (m, n)

so lie some side of the tangent

(0, 0) ⇒ –1 < 0

∴ (m, n) ⇒ am +bn – 1 < 0 ⇒ am +bn < 1

(m, n) & (a, b) can be equal

∴ am + bn ≤ 1

(m, n) & (a, b) can be negative

∴ |am + bn| ≤ 1

Sol.43 C

Chord of contant from (0, 0) & (g, f) are

gx + fy + c = 0

& gx + fy + g(x + g) + f(y + f) + c = 0

⇒ $2gx + 2fy + g^2 + f^2 + c = 0$

distance between C.O.C.'s

$$= \frac{\left|\frac{g^2+f^2+c-c}{2}\right|}{\sqrt{g^2+f^2}} = \frac{g^2+f^2-c}{2\sqrt{g^2+f^2}}$$

$\{\because g^2 + f^2 - c \ge 0\}$

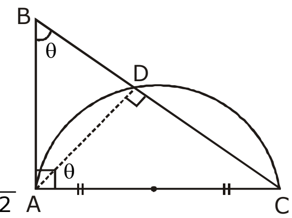
Sol.44 D

AD ⊥ BC

In ΔACD ⇒ $\frac{AD}{AC} = \sin\theta$ ....(i)

In ΔABD ⇒ $\frac{AD}{AB} = \cos\theta$ ....(ii)

$(i)^2 + (ii)^2$

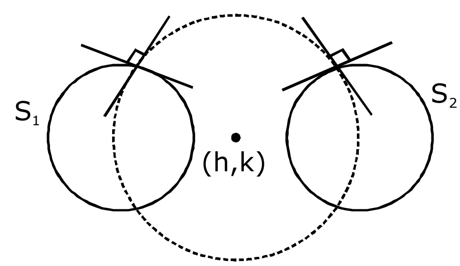


$\Rightarrow \frac{AD^2}{AC^2} + \frac{AD^2}{AB^2} = 1$

$\Rightarrow \frac{1}{AC^2} + \frac{1}{AB^2} = \frac{1}{AD^2}$

$\Rightarrow AC^2 = \frac{AB^2AD^2}{AB^2 - AD^2} \Rightarrow AC = \frac{AB.AD}{\sqrt{AB^2 - AD^2}}$

Sol.45 C



Let centre (h, k) & circle

$x^2 + y^2 + 2gx + 2fy + c = 0$

h = –g, k = –f

For S_1 : $g_1 = 2$, $f_1 = -3$, $c_1 = 9$,

For S_2 : $g_2 = -\frac{5}{2}$, $f_2 = 2$, $c_2 = -2$

∴ 2.g.2 + 2.f (–3) = c + 9

⇒ 4g – 6f = c + 9 ...(1)

& $2g\left(\frac{-5}{2}\right) + 2.f(2) = c - 2$

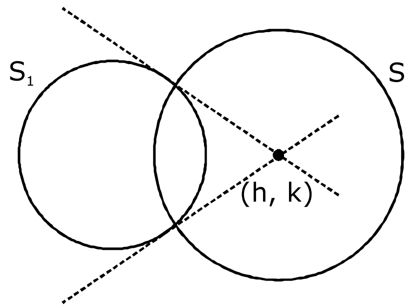
⇒ –5g + 4f = c – 2 ...(2)

Subtract (2) from (1)

– 9g + 10f = 11 ⇒ 9x – 10y + 11 = 0

Sol.46 A

Let point of intersection of tangents is (h, k) family of circle.



$x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0$

Common chord is $S - S_1 = 0$

$\Rightarrow -(\lambda + 6)x + (8 - 2\lambda)y - 2 = 0$

$\Rightarrow (\lambda + 6)x + (2\lambda - 8)y + 2 = 0$ ....(i)

C.O.C. from (h, k) to S_1: $x^2 + y^2 = 1$ is

$hx + ky = 1$ ....(ii)

(i) & (ii) are same equation

$$\frac{\lambda+6}{h} + \frac{2(\lambda-4)}{k} = \frac{2}{-1}$$

$\Rightarrow \lambda = -2h - 6, \quad \lambda = -k + 4$

$\therefore -2h - 6 = -k + 4$

$\Rightarrow 2h - k + 10 \Rightarrow$ Locus : $2x - y + 10 = 0$

Sol.47 D

Let equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

passes through (1, t), (t, 1) & (t, t)

$\Rightarrow 1 + t^2 + 2g + 2ft + c = 0$ ....(i)

$\Rightarrow t^2 + 1 + 2gt + 2f + c = 0$ ....(ii)

$\Rightarrow t^2 + t^2 + 2gt + 2ft + c = 0$ ....(iii)

by (i), (ii) & (iii) we get

$$g = -\frac{(t+1)}{2},\ f = -\frac{(t+1)}{2},\ c = 2t$$

$\therefore x^2 + y^2 - x(t + 1) - y(t + 1) + 2t = 0$

$(x^2 + y^2 - x - y) + t(-x - y + 2) = 0$

$\Rightarrow S + tL = 0$

Fixed point of intesection of S & L

$\therefore x^2 + y^2 = 2$

& $x + y = 2$

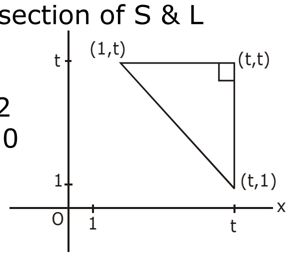
$\Rightarrow x^2 + (2 - x)^2 = 2$

$\Rightarrow 2x^2 - 4x + 2 = 0$

$\Rightarrow (x - 1)^2 = 0$

$\Rightarrow x = 1$ & $y = 1$

Point (1, 1)



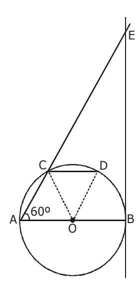
Sol.48 D

2CD = AB

CD = OC = OD = AC

$$\frac{AB}{AE} = \cos 60^\circ$$

$$AE = \frac{AB}{1/2} = 2AB$$



Sol.49 C

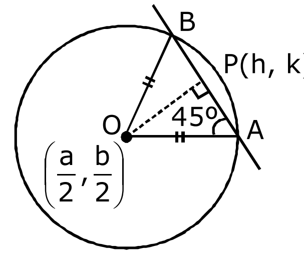
Given $x^2 + y^2 - ax - by = 0$

$$\text{Centre} \equiv \left(\frac{a}{2}, \frac{b}{2}\right),\ r = \frac{\sqrt{a^2+b^2}}{2}$$

In ΔOPA,

$$\Rightarrow \frac{OP}{OA} = \sin 45^\circ$$

$$\Rightarrow OP = \frac{OA}{\sqrt{2}}$$



$$\Rightarrow \frac{\sqrt{a^2+b^2}}{2\sqrt{2}} = \sqrt{\left(h-\frac{a}{2}\right)^2 + \left(k-\frac{b}{2}\right)^2}$$

$$\Rightarrow \frac{a^2+b^2}{8} = h^2 - ah + \frac{a^2}{4} + k^2 - bk + \frac{b^2}{4}$$

$$\Rightarrow h^2 + k^2 - ah - bk + \frac{a^2+b^2}{8} = 0$$

$$\Rightarrow x^2 + y^2 - ax - by + \frac{a^2+b^2}{8} = 0$$

Sol.50 A

Circle $x^2 + (y - b)^2 = b^2$

$\Rightarrow x^2 + y^2 - 2by = 0$

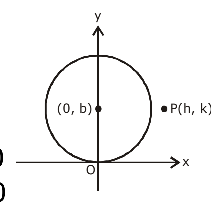
Polar w.r.t. circle P(h, k)

$\therefore hx + ky - b(y + k) = 0$

$\Rightarrow hx + y(k - b) - bk = 0$

Compair with

$\ell x + my + n = 0$



$$\Rightarrow \frac{\ell}{h} = \frac{m}{k-b} = \frac{n}{-bk}$$

$$\Rightarrow \ell = \frac{hn}{-bk}\ \&\ m = \frac{n(k-b)}{-bk}$$

$$\Rightarrow b = \frac{-hn}{\ell k}\ \&\ mbk + n(k - b) = 0$$

$$\therefore -mk\frac{hn}{\ell k} + n\left(k + \frac{hn}{\ell k}\right) = 0$$

$$\Rightarrow -\frac{mnh}{\ell} + \frac{n(k^2\ell + hn)}{k\ell} = 0$$

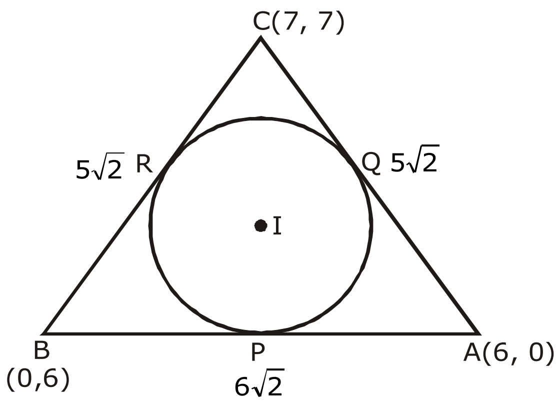
$\Rightarrow -mnhk + nk^2\ell + hn^2 = 0$

$\Rightarrow -mhk + k^2\ell + hn = 0$

$\Rightarrow h(mk - n) - \ell k^2 = 0$

$\Rightarrow x(my - n) - \ell y^2 = 0$

Sol.51 B



$\because$ P(3, 3)

$$\therefore I\left(\frac{6.(5\sqrt{2})+0+7.6\sqrt{2}}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}},\frac{0+6.(5\sqrt{2})+7(6\sqrt{2})}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}}\right)$$

$$I\left(\frac{9}{2},\frac{9}{2}\right),\ r = IP = \sqrt{\left(\frac{9}{2}-3\right)^2+\left(\frac{9}{2}-3\right)^2} = \frac{3}{\sqrt{2}}$$

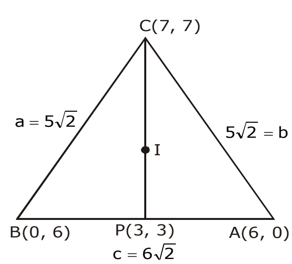
$$\Rightarrow \left(x-\frac{9}{2}\right)^2 + \left(y-\frac{9}{2}\right)^2 = \frac{9}{2}$$

$$\Rightarrow x^2 + y^2 - 9x - 9y + \frac{81}{2} - \frac{9}{2} = 0$$

$$\Rightarrow x^2 + y^2 - 9x - 9y + 36 = 0$$

Aliter

I divdies CP in the ratio $\frac{a+b}{c}$



$$= \frac{10\sqrt{2}}{6\sqrt{2}} = \frac{5}{3}$$

$$I \equiv \left(\frac{7.3+3.5}{3+5},\frac{7.3+3.5}{3+5}\right) \equiv I\left(\frac{9}{2},\frac{9}{2}\right)$$

equation of circle

Sol.52 B

AC = 2 = AB = BC = CA = AD

$OB = \sqrt{2^2-1} = \sqrt{3}$

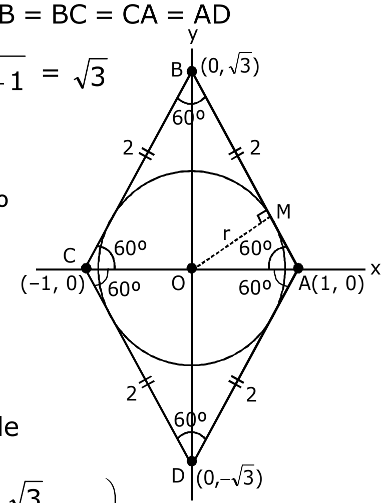
In ΔOAM,

$\frac{r}{OA}$ sin 60º

$\Rightarrow r = \frac{\sqrt{3}}{2}$

Any point on the circle

$P\left(\frac{\sqrt{3}}{2}\cos\theta,\frac{\sqrt{3}}{2}\sin\theta\right)$



$$|PA|^2 = \left(\frac{\sqrt{3}}{2}\cos\theta-1\right)^2+\left(\frac{\sqrt{3}}{2}\sin\theta\right)^2 = \frac{3}{4}+1-\cos\theta$$

$$|PB|^2 = \left(\frac{\sqrt{3}}{2}\cos\theta\right)^2+\left(\frac{\sqrt{3}}{2}\sin\theta-\sqrt{3}\right)^2 = \frac{3}{4}+3-3\sin\theta$$

$$|PC|^2 = \left(\frac{\sqrt{3}}{2}\cos\theta+1\right)^2+\left(\frac{\sqrt{3}}{2}\sin\theta\right)^2 = \frac{3}{4}+1+\sqrt{3}\cos\theta$$

$$|PD|^2 = \left(\frac{\sqrt{3}}{2}\cos\theta\right)^2+\left(\frac{\sqrt{3}}{2}\sin\theta+\sqrt{3}\right)^2 = \frac{3}{4}+3+3\sin\theta$$

---

sum = $4 \cdot \frac{3}{4} + 8 = 11$

Sol.53 A

$x^2 + y^2 < 25$

on x-axis & y-axis 4 × 4 + 1 = 17

x = 1, y = 1, 2, 3, 4

x = 2, y = 1, 2, 3, 4

x = 3, y = 1, 2, 3

x = 4, y = 1, 2

In I^s quadrant 13

In all quadrant = 13 × 4 = 52

No. of points = 52 + 17 = 69